# Tax Reform with Endogenous Borrowing Limits and Incomplete Asset Markets
## (Preliminary and Incomplete)

**Árpád Ábrahám**[*]

University of Rochester

**Eva Cárceles-Poveda**[†]

SUNY at Stony Brook

December 9, 2008

ABSTRACT. This paper studies different income tax reforms in an infinite horizon economy with a progressive labor income tax code, incomplete makets an endogenous borrowing constraints on capital holdings. In particular, it assumes that households can break their trading arrangements by going into financial autarky, in which case they are excluded from future asset trade. The endogenous limits are then determined at the level at which households are indifferent between defaulting and paying back their debt. These limits are significantly different from zero and they get looser with a higher labour income, a property that is consistent with US data on credit limits. The reforms we study are all revenue neutral and they eliminate capital income taxes but they differ in the changes to the labor income tax code. Our results illustrate that a successful reform has to combine the increase in average labor taxes with an increase in the progressivity of the labor income tax code. On the one hand, this reduces the disposable income of the rich, leading to lower savings and to a lower aggregate capital. On the other hand, it allows the poor and middle income households to supply less labor and consume and save more after the reform, increasing the aggregate welfare both in the long run an throughout the transition.

*Keywords: Endogenous Borrowing Constraints, Incomplete Markets, Production, Tax Reform.*

*JEL Classification: E23, E44, D52*

———————————

[*]*Correspondence:* Department of Economics, University of Rochester, Harkness Hall, P.O.Box 270156, Rochester NY 14627. *E-mail:* `aabraha2@mail.rochester.edu`.

[†]*Correspondence:* Department of Economics, State University of New York, Stony Brook NY-11794-4384 and IAE-CSIC. *Email:* ecarcelespov@notes.cc.sunysb.edu. *Web:* http://ms.cc.sunysb.edu/~ecarcelespov/.

# 1. INTRODUCTION

This paper studies tax reform in an infinite horizon incomplete markets model with production where borrowing constraints on asset holdings are endogenous. We study the desirability of the revenue neutral elimination of capital income taxes. In our economy, capital income taxes discourage capital accumulation and in principle their elimination can lead to long run welfare gains. However, the only way to recover the lost revenue for the government is to adjust the labor income tax system. We study what changes in labor income taxes can lead to an increase in aggregate welfare and can gain political support.

In order to be able to study the welfare effect of a realistic tax reform, we need to have a model with a realistic wealth distribution. The fact that there is a significant proportion of individuals in debt in the data implies that a realistic model of incomplete markets should also be able to generate enough borrowing. Clearly, these two aspects are interrelated through the borrowing constraints, since they are one of the key determinants of the (equilibrium) level of debt and in general of the wealth distribution in these types of economies. In the present paper, we determine these constraints endogenously and we explicitly take this into account by calibrating the model so that the distribution of assets and the amount of debt matches the one in the data.

To endogeneize the borrowing limits, we introduce the possibility of default on financial liabilities. In particular, we assume that households can break their trading contracts every period. In this case, individual liabilities are forgiven and agents are excluded from future asset trade forever. The endogenous trading limits are then set at the level at which households are indifferent between honoring their debt and defaulting.

An appealing property of endogeneizing the borrowing limits becomes more apparent when we consider policy applications such as the reforms we study. In a framework in which the equilibrium allocations exhibit imperfect risk sharing, changes in economic policy typically affect the wealth distribution. In the presence of limited commitment, these changes also affect the relative value of default and consequently the endogenous borrowing constraints. This is particularly important in models with capital accumulation, generating quantitatively important general equilibrium effects that interact with the borrowing limits. For this reason, we compare our results with endogenous limits with an economy where the limits are fixed at zero.

Using the calibrated economy, we study the effect of different tax reforms quantitatively both in the long run (comparing steady states) and in the short run by analyzing the transitional dynamics. Our main result is that the only reform which has a chance to increase aggregate welfare both in the short and long run is a reform in which the elimination of capital income taxes an the subsequent increase in average labor income taxes are accompanied by an increase in the progressivity of the labor income tax system. The key is that reforms which finance the decrease in capital taxes only through an increase in the average tax rate (or by decreasing the progressivity of the tax system), cannot be succesful, as they put too high burden on the asset poor indviduals. These agents benefit little from the higher (after tax) interest rate but loose considerably because of higher labor taxes and a lower disposable income. A reform that increases the progressivity of the labor income tax code can therefore

gain political support and increase aggregate welfare, although it taxes high income people more heavily and for that reason leads to a lower aggregate capital eventually. This implies that the reform increases welfare by decreasing inequality. In fact, the elimination of capital taxes increases the savings of the low income households, while this is more than offset by the reduction of asset accumulation of the high income individuals.

When the borrowing limits are endogenous, it is important to note that the reform makes default more attractive for borrowers, as interest rates become higher. We show, that in equilibrium, this will lead to tighter limits. In turn, the tighter limits hurt people who are already borrowing constrained, as they have to decrease their debt. The full transition dynamics of this case has not been calculated yet, but this effect is going to be part of the analysis in future versions.

Our work builds a bridge between two important strands of literature. First, it contributes to an increasingly growing literature in which a number of authors have introduced limited enforceability of risk-sharing contracts in models with complete markets, implicitly resulting in agent and state specific trading constraints. Among others, Kehoe and Levine (1993), Alvarez and Jermann (2000, 2001) and Krueger and Perri (2005) introduce these type of limits in exchange economies, whereas Kehoe and Perri (2002, 2004) study a production economy where investors are interpreted as countries. Since the lack of commitment leads to equilibrium allocations that exhibit imperfect risk sharing, these models are labelled endogenous incomplete market economies. Apart from the fact that this literature does not characterize the endogenous borrowing limits, the imperfect risk sharing result may not be robust to the introduction of capital accumulation in closed economy models. For example, Ábrahám and Carceles-Poveda (2007b) show that the equilibrium of a two agent model with endogenous production exhibits full risk sharing in the long run for standard parameterizations. Since the implications of models with full or close to full risk sharing are clearly at odds with the data, this provides a strong motivation to study limited commitment in economies with incomplete markets, where risk sharing is always limited. While the number of assets traded is still exogenous in this case, the presence of limited commitment endogenizes the amount that households can borrow. In this sense, the degree of market incompleteness becomes partially endogenous. The present paper uses the structure introduced in Ábrahám and Carceles-Poveda (2007c), but adds endogenous labor supply and study a more sophisticated tax reform both in the short and long run.

Second, our work is related to the recent literature studying the welfare effects of capital income taxation in a context with heterogeneous agents. For example, Aiyagari (1995) studies the optimal capital income tax in a model with incomplete markets and no borrowing. In contrast to the seminal papers of Chamley (1986) and Judd (1985), who show that the optimal long run capital income tax is zero for a wide class of infinite horizon models with complete markets, the author shows that the optimal long run capital income tax is always strictly positive. Further, in a model with no borrowing and a more realistic calibration, Domeij and Heathcote (2004) find that eliminating capital income taxes in a setting with no borrowing and flat tax rates may be welfare improving in the long run, while it decreases welfare in the short run. We confirm the results of Domeij and Heathcote (2004) with a

more general tax system for the case in which the tax reform is accompanied by an increase in the average tax rate. However we also show that the results are different if we consider a setting with endogenous limits and if we can change the progressivity of the labor income tax system. In addition, Conesa and Krueger (2007, 2008) study tax reforms in the presence of a progressive labor income tax code in a setting with overlapping generations and no borrowing. In contrast, our setting is an infinite horizon economy and it allows for endogenous borrowing limits.

Finally, we should note that the presence of endogenous trading limits considerably complicates our computations, since we have to extend usual policy (or value) iteration algorithm to incorporate a state dependent and non rectangular grid for some of the endogenous states, introducing an additional fixed point problem. In spite of the computational difficulties, however, the methods developed in the present work could be fruitfully applied to study a wide set of interesting incomplete market models with endogenous limits. An example is the recent work by Bai and Zhang (2005), where the authors show that such an economy can account better for the observed cross country correlations of savings and investment rates than the complete markets counterpart. In addition, our results also suggests that other types of fiscal policy and social insurance programs can have significant effects on the level of the endogenous trading constraints. Given this, a welfare analysis of any policy reform should take these effects into account.

The rest of the paper is organized as follows. Section 2 presents the general model with incomplete markets. Section 3 presents the calibration and numerical solution of the benchmark model and Section 4 analyzes the welfare implications of a tax reform in the long run while Section 5 does the same throughout the transition for endogenous and exogenous borrowing limits. Finally, Section 6 summarizes and concludes.

## 2. The Model

We consider an infinite horizon economy with endogenous production, idiosyncratic labor productivity shocks and sequential asset trade subject to portfolio restrictions. The economy is populated by a government, a representative firm and a continuum (measure 1) of infinitely lived households that are indexed by $i \in I$.

**Households.** Households are endowed with one unit of time and they can use it to either supply labor to the firm or to consume leisure. Preferences over sequences of consumption $c_i \equiv \{c_{it}\}_{t=0}^{\infty}$ and leisure $1 - l_i \equiv \{1 - l_{it}\}_{t=0}^{\infty}$ are assumed to be time separable:

$$U(c_i, 1 - l_i) = E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_{it}, 1 - l_{it}\right), \tag{1}$$

where $\beta \in (0, 1)$ is the subjective discount factor and $E_0$ denotes the expectation conditional on information at date $t = 0$. We assume that the period utility function $u : \mathbb{R}_+^2 \to \mathbb{R}$ is strictly increasing and continuously differentiable in both arguments.

Each period, household $i \in I$ receives a stochastic labour productivity shock $\epsilon_i$. This shock is is i.i.d. across households and it follows a Markov process with transition matrix $\Pi(\epsilon'|\epsilon)$ and $S_\epsilon$ possible values that are assumed to be strictly positive. A household working

$l_{it}$ hours has a pre-tax labor income of $y_{pt} = w_t l_{it} \epsilon_{it}$, where $w_t$ is the wage rate per efficiency unit of labor. Labor income taxes are assumed to be progressive and they are set by the government according to the function $T_l(y_{pt})$.

To insure against their idiosyncratic labor income risk, we assume that households can trade (borrow or save) in physical capital, whose interest income is subject to a proportional capital income tax $\tau_k$.[1] The after-tax gross return on capital is therefore equal to $1 + r_t(1 - \tau_k(k_{it}))$, where $k_{it}$ represents the beginning of period individual capital holdings. We assume that only savers pay taxes on interest income. Given this, capital income taxes depend on the level of assets in the following way:

$$\tau_k(k_{it}) = \begin{cases} \boldsymbol{\tau_k} & \text{if } k_{it} \geq 0 \\ 0 & \text{if } k_{it} < 0 \end{cases}$$

The households' budget constraint can be expressed as:

$$c_{it} + k_{it+1} = w_t l_{it} \epsilon_{it} - T_l(w_t l_{it} \epsilon_{it}) + (1 + r_t(1 - \tau_k(k_{it}))) k_{it}. \tag{2}$$

At each date, household $i \in I$ also faces a possibly endogenous and state-dependent trade restriction on the end of period capital holdings $k_{it+1}$. Throughout the paper, we assume that households cannot commit on the trading contracts and we determine the borrowing constraint endogenously at the level that prevents default in equilibrium. In case of default, we assume that individual liabilities are forgiven and households are excluded from future asset trade. Households can continue supplying labor to the firm and this implies that their only source of income from the default period is their labor income. Following Livshits, MacGee and Tertilt (2006), we also assume that there is an additional penalty $\lambda$ that reduces labour income by $(1 - \lambda)$ after default. This penalty can be interpreted as a reduced form for different monetary and non monetary costs of defaulting, such as the fraction of income that is garnished by creditors, the utility (stigma), the fixed monetary costs of filing, and the increased cost of consumption.[2]

**Production.** At each date, the representative firm uses capital $K_t \in \mathbb{R}_+$ and labor $L_t \in (0, 1)$ to produce a single good $y_t \in \mathbb{R}_+$ with the constant returns to scale technology:

$$y_t = Af(K_t, L_t), \tag{3}$$

where $A$ is a technology parameter that represents total factor productivity. The production function $f(\cdot, \cdot) : \mathbb{R}_+^2 \to \mathbb{R}_+$ is assumed to be continuously differentiable on the interior of its domain, strictly increasing, strictly concave in $K$ and homogeneous of degree one in $K$ and $L$. Capital depreciates at the rate $\delta$ and we denote total output including undepreciated capital by:

$$F(K_t, L_t) = Af(K_t, L_t) + (1 - \delta)K_t. \tag{4}$$

---

[1] This framework can be easily extended to the presence of trade in more than one asset.

[2] This punishment for default resembles the bankruptcy procedures under Chapter 7. Under this procedure, households are seized from any positive asset holdings but can keep at least part of their labour income. Whereas they are allowed to borrow after some periods, this becomes considerably more difficult and costly because their credit rating deteriorates significantly.

Each period, the firm rents capital and labor to maximize period profits:

$$F(K_t, L_t) - w_t L_t - r_t K_t, \qquad (5)$$

leading to the following first order conditions:

$$w_t = f_L(K_t, L_t) \qquad (6)$$

$$r_t = f_K(K_t, L_t) - \delta. \qquad (7)$$

**Government and Market Clearing.** At each period $t$, the government consumes the amount $G_t$ and it taxes individual labor income according to $T_l(\cdot)$ and individual capital income at the rate $\tau_k$. The government is assumed to have a balanced budget. As usual, the labor and asset market clearing conditions require that the sum of individual labor supply times the productivity shock is equal to the total labor supply, while the sum of individual capital holdings are equal to the aggregate capital stock. Further, the good's market clearing condition requires that the sum of investment and aggregate consumption, including household and government consumption, is equal to the aggregate output.

**Recursive Competitive Equilibrium.** In the present framework, the aggregate state of the economy is given by the joint distribution $\Psi$ of consumers over individual capital holdings $k$ and idiosyncratic productivity status $\epsilon$. Further, households perceive that $\Psi$ evolves according to:

$$\Psi' = \Gamma[\Psi],$$

where $\Gamma$ represents the transition function from the current aggregate state into tomorrow's wealth-productivity distribution. Since the individual state vector includes the individual labour productivity and capital holdings $(\epsilon, k)$, the relevant state variables for a household are summarized by the vector $(\epsilon, k; \Psi)$.

Using this notation, the outside option or autarky value $V$ of a household with income shock $\epsilon$ can be expressed recursively as:

$$V(\epsilon; \Psi) = u(y^{au}(\epsilon; \Psi)(1 - \lambda)) + \beta \sum_{\epsilon'} \Pi(\epsilon'|\epsilon) V(\epsilon'; \Gamma[\Psi]). \qquad (8)$$

where $y^{au}(\epsilon; \Psi) = w(\Psi) l^{au}(\epsilon) \epsilon - T_l(w(\Psi) l^{au}(\epsilon) \epsilon)$ and $l^{au}(\epsilon)$ are the disposable income and the optimal labor choice in autarky respectively.

Equation (8) reflects that the autarky value is a function of the wealth-productivity distribution. Note that this is in contrast with some of the literature with complete markets and no commitment, where $V$ is exogenous (see e.g. Alvarez and Jermann (2000, 2001)). As we will see later, this is due to the fact that the distribution determines aggregate capital accumulation, which in turn determines future wages and therefore the future value of financial autarky. On the other hand, since individual liabilities are forgiven upon default, the autarky value is not a function of the individual capital holdings. Note also that the expression in (8) implicitly assumes that the aggregate state of the economy follows the same law of motion $\Gamma[\Psi]$ if one of the agents defaults. This is correct in the presence of a continuum of agents,

6

since an individual deviation does not influence the aggregate variables and no one defaults in equilibrium.

We are now ready to define the recursive competitive equilibrium. Since the aggregate labor supply is constant due to a law of large numbers, factor prices only depend on the aggregate production factors (capital and labor) and we therefore write $w(\Psi) = w(K, L)$ and $r(\Psi) = r(K, L)$ in what follows. On the other hand, in order to guarantee balanced budget, the tax policy function depends on the whole distribution of individuals, hence we denote it by $T_l(y; \Psi)$.

**Definition 2.1:** Given a transition matrix $\Pi$ and some initial distribution of shocks $\epsilon_0 \equiv (\epsilon_{i0})_{i \in I}$ and asset holdings $k_0 \equiv (k_{i0})_{i \in I}$, a *recursive competitive equilibrium* relative to the capital income tax rate $\tau_k$ and the labor income tax function $T_l(\cdot; \Psi) : \mathbb{R}_+ \to \mathbb{R}_+^\infty$ and borrowing limits $\underline{k}(\epsilon; \Psi)$ is defined by a law of motion $\Gamma$, a vector of factor prices $(r, w) = (r(K, L), w(K, L))$, a government consumption $G$, value functions $W = W(\epsilon, k; \Psi)$ and $V = V(\epsilon; \Psi)$, and individual policy functions $(c, k') = (c(\epsilon, k; \Psi), l(\epsilon, k; \Psi), k(\epsilon, k; \Psi))$ such that:

(i) *Utility Maximization:* For each $i \in I$, $W$ and $(c, l, k')$ solve the following problem given $k_0$, $\epsilon_0$, $\Pi$, $\Gamma$, $T_l$ and $\tau_k$ and $(r, w)$:

$$W(\epsilon, k; \Psi) = \max_{c, k'} \left\{ u(c, 1 - l) + \beta \sum_{\epsilon'} \Pi(\epsilon'|\epsilon) W(\epsilon', k'; \Psi') \right\} \quad (9)$$

s.t. $c + k' = w(K, L)l\epsilon - T_l(w(K, L)l\epsilon; \Psi) + (1 + r(K, L)(1 - \tau_k(k))) k$

$$\Psi' = \Gamma[\Psi]$$

$$c \geq 0, 0 \leq l \leq 1$$

$$k' \geq \underline{k}(\epsilon'; \Psi') \text{ for all } \epsilon'|\epsilon \text{ with } \Pi(\epsilon'|\epsilon) > 0.$$

(ii) *Profit Maximization:* Factor prices satisfy the firm's optimality conditions, i.e., $w(K, L) = f_L(K, L)$ and $r(K, L) = f_K(K, L) - \delta$.

(iii) *Balanced Budget:* The government budget constraint is satisfied, i.e.,

$$G = \int T_l(w(K, L)l\epsilon; \Psi) \, d\Psi(\epsilon, k) + r(K, L)\tau_k \widehat{K}, \text{ where}$$

$$\widehat{K} = \int_{k \geq 0} k d\Psi(\epsilon, k)$$

is the sum of capital holdings of those who hold non-negative assets.

(iv) *Market Clearing:*

$$\int k(\epsilon, k; \Psi) d\Psi(\epsilon, k) = K'$$

$$\int l(\epsilon, k; \Psi)\epsilon d\Psi(\epsilon, k) = L$$

$$\int [c(\epsilon, k; \Psi) + k(\epsilon, k; \Psi)] \, d\Psi(\epsilon, k) + G = F(K, L) + (1 - \delta)K$$

(v) *Consistency:* $\Gamma$ is consistent with the agents' optimal decisions, in the sense that it is generated by the optimal decision rules and by the law of motion of the shock.

(vi) *No default:* $\underline{k}(\epsilon; \Psi)$ is such that individuals are indifferent between trading and going into autarky, i.e.,

$$\underline{k}(\epsilon; \Psi) = \{\underline{k} : W(\epsilon, \underline{k}; \Psi) = V(\epsilon; \Psi)\}. \tag{10}$$

where

$$V(\epsilon; \Psi) = \max_{c,l} \left\{ u(c, 1-l) + \beta \sum_{\epsilon'} \Pi(\epsilon'|\epsilon) V(\epsilon'; \Psi') \right\}$$

$$\text{s.t. } c = w(K, L)l\epsilon - T(w(K, L)l\epsilon)$$

$$\Psi' = \Gamma[\Psi]$$

$$c \geq 0, \, 0 \leq l \leq 1$$

Several remarks are worth noting. First, as reflected in condition (i), households are only allowed to hold levels of individual capital that are above a state-dependent lower bound for each continuation state with positive probability next period. This implies that the effective limit on capital holdings $\kappa(\epsilon; \Psi)$ faced by a household is the tightest among these state-dependent lower bounds. Using the recursive notation, the effective borrowing constraints can therefore be expressed as:[3]

$$k' \geq \kappa(\epsilon; \Psi) \equiv \sup_{\epsilon':\Pi(\epsilon'|\epsilon)>0} \left\{ \underline{k}\left(\epsilon'; \Gamma[\Psi]\right) \right\}. \tag{11}$$

Second, the definition of the state-dependent lower bounds in (10) implies that we can think about $\underline{k}(\epsilon; \Psi)$ as a state-dependent default threshold, since it represents the level of capital holdings such that households are indifferent between defaulting and paying back their debt. Clearly, condition (vi) implies that we only consider equilibria where the trading limits are such that default is not possible. Whereas there are many borrowing limits that prevent default in equilibrium, we consider the loosest possible ones of such limits. In other words, we study the economy with limits that are not too tight, in the sense that they satisfy (10) and (11).

The following proposition shows the existence of a unique lower bound $\underline{k}(\epsilon; \Psi)$ satisfying equation (10). The proof is relegated to the Appendix.

**Proposition 2.1.** *If u is unbounded below, equation (10) defines a unique, non-positive and finite default threshold $\underline{k}(\epsilon; \Psi)$ for every $\epsilon$ and $\Psi$.*

The proof of this proposition follows Zhang (1997a, 1997b), who characterizes the default thresholds in exchange economies with exogenous labor supply. In particular, the existence of the default thresholds established by Proposition 2.1 is a consequence of the fact that $V(\epsilon; \Psi)$ is finite, while $W(\epsilon, k; \Psi)$ goes to minus infinity as $k$ goes to the natural borrowing limit. In addition, uniqueness simply follows from the fact that $V(\epsilon; \Psi)$ does not depend on

---

[3]If the probability of all future shock realizations is strictly positive for any given shock, the effective limit faced by the households will not be a function of the current shock, since the trading restriction has to be satisfied for all possible continuation states. This will not be the case, however, in our calibrated example.

$k$ while $W(\epsilon, k; \Psi)$ is strictly increasing in $k$. An important implication of uniqueness is the fact that the value of staying in the trading arrangement is always higher than the autarky value if the capital holdings are above the default threshold, that is,

$$W(\epsilon, k; \Psi) \geq V(\epsilon; \Psi) = W(\epsilon, \underline{k}; \Psi) \text{ for } \forall k \geq \underline{k}(\epsilon; \Psi).$$

The fact that the thresholds are finite is a consequence of the fact that $V(\epsilon; \Psi)$ is finite. Finally, the equilibrium default thresholds and effective limits have to be clearly non-positive. Intuitively, note that agents would not default with a positive level of asset holdings, since they could then afford a higher current consumption than in autarky and at least as high of a life-time utility as in autarky from next period on by paying back their debt.

An important property of the endogenous borrowing thresholds $\underline{k}(\epsilon; \Psi)$ will be their dependence on the labor income shock. We can further characterize this dependence if we assume differentiability of both the trading and autarky values and the following continuous process for the idiosyncratic labor income shock:

$$\log(\epsilon') = \mu_\epsilon + \rho_e \log(\epsilon) + \varepsilon'_\epsilon \text{ with } \varepsilon'_\epsilon \sim N(0, \sigma_\epsilon^2).$$

Under these assumptions, we can express the effects of a change in $\epsilon$ by differentiating equation (10), obtaining that:

$$\frac{\partial \underline{k}(\epsilon; \Psi)}{\partial \epsilon} = -\frac{W_\epsilon(\epsilon, \underline{k}; \Psi) - V_\epsilon(\epsilon; \Psi)}{W_k(\epsilon, \underline{k}; \Psi)}. \tag{12}$$

In the previous equation, $W_\epsilon(\epsilon, \underline{k}; \Psi)$ and $V_\epsilon(\epsilon; \Psi)$ represent the derivatives of the two value functions, evaluated at $\underline{k}$, with respect to the income shock $\epsilon$. Similarly, $W_k(\epsilon, \underline{k}; \Psi)$ represents the derivative of the trading value, evaluated at $\underline{k}$, with respect to $k$.

Since more individual capital holdings (ceteris paribus) expand the budget sets, and because the utility function is strictly increasing, it follows that $W_k(\epsilon, \underline{k}; \Psi) > 0$. Given this, the sign of the previous derivative is determined by wether a change in income increases the trading value $W$ more or less than the autarky value $V$. If the trading value increases more than the autarky value after an increase in the income shock, the derivative will be negative. In this case, a higher income will lead to looser default thresholds. As shown by Ábrahám and Carceles-Poveda (2007c), if households do not derive utility from leisure and labor taxes are flat, the higher is the productivity shock of an agent, the looser are the default thresholds, i.e. $\frac{\partial \underline{k}(\epsilon; \Psi)}{\partial \epsilon} \leq 0$. As our numerical results will show, this property is robust to the presence of a labor-leisure decision and progressive labor income taxes. Given that the ability to borrow is a positive function of income in the data (see Ábrahám and Carceles-Poveda (2007c)), this result is a desirable property of the present setting. Of course, a key aspect for obtaining this result is that markets are incomplete.

Finally, it is important to note that the default thresholds are very closely related to the endogenous borrowing limits on Arrow securities that are defined in the literature with complete markets and limited commitment. Among others, Alvarez and Jermann (2000) and Ábrahám and Carceles (2007a) define these limits in endowment and production economies, respectively.

## 3. Quantitative Results

This section first studies the stationary distribution of a calibrated version of the model described above. Note that, in the steady state, all aggregate variables, including the asset distribution, government consumption, taxes, the aggregate capital and factor prices are constant. First, we discuss the calibration and solution method for the benchmark economy. Next, we study the properties of the endogenous borrowing limits, particularly the relationship between these limits and income. Furthermore, the equilibrium allocations are compared to the ones resulting when the limits are exogenously fixed at zero, which is the typical assumption in the incomplete markets literature (see e.g. Aiyagari (1994, 1995) or Krusell and Smith (1997, 1998)) and in the literature studying tax reform (e.g. Domeij and Heathcote, 2004 and Conesa and Krueger, 2007).

**3.1. Calibration and Solution Method.** One of the main aims of the calibration is that the model steady state matches the earnings and wealth distribution in the US. In addition, we target several aggregate statistics, such as the labor share, the capital output ratio and the interest rate.

The time period is assumed to be one year and the depreciation rate is therefore set to $\delta = 0.08$. The production function is Cobb Douglas, $f(K,L) = AK^\alpha L^{1-\alpha}$. The capital share is set to $\alpha = 0.36$ to match the labor share of 0.64 in the US data. Further, the technology parameter $A$ is chosen so that output is equal to one in the steady state of the deterministic economy.

For preferences, we assume the Cobb Douglas function $u(c, 1-l) = \frac{\left[c^\gamma (1-l)^{1-\gamma}\right]^{1-\sigma}}{1-\sigma}$, where $\gamma$ determines the relative importance of consumption and $\sigma$ is the level of risk aversion. We set $\sigma = 4$ and calibrate $\gamma$ so that the average share of time worked is $1/3$.

We want the income tax code to be a good approximation of the one in US. To achieve this, we assume a flat capital income tax of $\tau_k = 0.4$, which is very close to the value found by Domeij and Heathcote (2004) using the method of Mendoza et. al (1994). Further, we assume progressive labor income taxes. In particular, if $y_p = wl\epsilon$ represents taxable income, the labor income tax is represented by the function:

$$T_l(y_p) = \kappa_0 \left( y_p - \left( y_p^{-\kappa_1} + \kappa_2 \right)^{-\frac{1}{\kappa_1}} \right)$$

where $(\kappa_0, \kappa_1, \kappa_2)$ are parameters. This functional form was originally proposed by Gouveia and Strauss (1994), who estimated the function for the US income tax code. Subsequently, it has been analyzed by several authors such as Castaneda et al (1999), Smyth(2005), Conesa and Krueger (2006,2008) and Garriga and Schlagenhauf (2008). Note that, in the previous function, the average labor income tax rate is governed by the parameter $\kappa_0$, while $\kappa_1$ governs the degree of progressivity. In particular, when $\kappa_1 \to 0$, the system becomes a flat tax, while $\kappa_1 > 0$ and $\kappa_1 < 0$ imply that the tax system is progressive and regressive respectively.

Gouveia and Strauss estimated the parameters of the above tax function for the US and they find that $\kappa_0 = 0.258$ and $\kappa_1 = 0.768$. In the benchmark version of the model, we maintain these values for $\kappa_0$ and $\kappa_1$ and we calibrate $\kappa_2$ to ensure government budget balance, with a target government to output ratio of $\frac{G}{Y} = 0.17$. This implies a value of

$\kappa_2 = 1.05$ in our benchmark economy. The government to output ratio is kept constant across all our experiments.

Table 1 describes the earnings process. It displays the shock values, the stationary distribution and the transition matrix. As reflected by the table, we have used a seven state income process. The process, which is similar to the ones used by Diaz et. al (2003) and Davila et. al (2007), is calibrated so that it generates a Gini coefficient for earnings of 0.6 (and thus the same concentration for income as in the data), as well as a realistic wealth distribution in the benchmark steady state. In particular, the income process, together with a discount factor of $\beta = 0.91$ and the default penalty of $\lambda = 0.075$, matches a capital output ratio of around 3, an interest rate of around 4% and the total financial assets hold by the lowest and highest quintiles of the US wealth distribution.[4]

Table 1: Earnings Process

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| $\epsilon =$ | 0.1805 | 0.3625 | 0.8127 | 1.8098 | 3.8989 | 8.4002 | 18.0980 |
| $\Pi^* =$ | 0.3173 | 0.2231 | 0.3128 | 0.0719 | 0.0453 | 0.0245 | 0.0051 |
| $\Pi(\epsilon'|\epsilon) =$ | 0.9687 | 0.0313 | 0 | 0 | 0 | 0 | 0 |
| | 0.0445 | 0.8620 | 0.0935 | 0 | 0 | 0 | 0 |
| | 0 | 0.0667 | 0.9180 | 0.0153 | 0 | 0 | 0 |
| | 0 | 0 | 0.0666 | 0.8669 | 0.0665 | 0 | 0 |
| | 0 | 0 | 0 | 0.1054 | 0.8280 | 0.0666 | 0 |
| | 0 | 0 | 0 | 0 | 0.1235 | 0.8320 | 0.0445 |
| | 0 | 0 | 0 | 0 | 0 | 0.2113 | 0.7887 |

Table 2 contains information about the wealth distribution in our benchmark model and in the 2004 Survey of consumer finances. Since the present paper is about unsecured credit, we have tried to match some key moments of the distribution of net financial assets. In contrast, most of the macroeconomic literature focuses on the wealth distribution based on the net worth, defined as the difference between total assets and total liabilities. When calculating net financial assets, we exclude the value of residential property, vehicles and direct business ownership from the assets, and the value of secured debt due to mortgages and vehicle loans from the liabilities. This level of assets represents better the amount of liquid assets that households can use to smooth out income shocks. Moreover, both residential properties and vehicles can be seen as durable consumption as much as investment.

As we see in the Table, according to the 2004 Survey of Consumer Finances, the lowest quintile of the wealth distribution, as measured by net financial assets, held -1.55% of total financial wealth, whereas 91.19 percent was held by the highest quintile. Our model matches this aspect of the distribution very well, since the assets held by the lowest and highest

---

[4]As discussed in Livshits, MacGee and Tertilt (2006), bankruptcy filers face several types of punishment. Apart from the fact that filers cannot save or borrow, a fraction of earnings is garnished by creditors in the three year period of filing. In addition, there are utility (stigma) and fixed monetary costs of filing that imply that a fraction of consumption may be lost. To match key observations regarding the evolution of bankruptcy filings in the last decades, the authors choose a garnishment rate of 0.319 and set the other costs to zero. Given this $\lambda = 0.1$ does not seem to be excessively high.

quintiles in the model are -1.50 and 90.48 respectively. Further, we also match reasonably well the asset holdings of the three medium quintiles. On the other hand, 19.97% of the population was in debt in the data, while our model implies that the number of people in debt is around 35.9%. In other words, the model somewhat overestimates the population in debt.[5] Nevertheless, most models studying tax reforms in a similar framework, such as Aiyagari (1995) and Domeij and Heathcote (2004), assume no borrowing. As we show in the next section, this assumption may have important limitations, in the sense that a model with endogenous borrowing limits can lead to very different conclusions regarding the welfare effects of tax reforms. In addition, a model with no borrowing cannot capture the fact that there is a substantial percentage of people in debt.

Table 2: The Wealth Distribution in the Benchmark Model and in the Data

|  | | Quintiles | | | | |
| --- | --- | --- | --- | --- | --- | --- |
| Economy | Q1 | Q2 | Q3 | Q4 | Q5 | % in Debt |
| Benchmark (pre-reform) | -1.50 | -0.72 | 3.16 | 8.57 | 90.48 | 35.96 |
| USA (net financial assets) | -1.55 | 0.09 | 1.61 | 8.66 | 91.19 | 19.97 |
| USA (net worth) | -0.18 | 1.13 | 4.37 | 17.10 | 82.90 | 7.14 |

**Solution Method.** To find the solution, we use a policy function iteration algorithm. Solving the model with endogenous trading limits involves several computational difficulties. First, our state space is endogenous, a problem that we address by incorporating an additional fixed point problem to find the state-dependent limits on the individual capital holdings. This also implies that our policy functions have to be calculated over a non-rectangular grid. Further, given that the limits in our model are endogenously determined at the level where the value function from staying in the contract is at least as large as the autarky value, it becomes clear that a good approximation of the value functions close to the limits is needed to obtain reliable results. To address this issue, we use a relatively high number of grid points, we interpolate the policy and value functions over this grid and we allow the limits to take values between grid points as well. In order to speed up the solution procedure we update the interest rate and the borrowing limits simultaneously.

**3.2. Results for the Benchmark Economy.** We summarize the aggregate statistics for the benchmark economy in Table 3. The two columns display some of the steady state statistics in the benchmark economy and the economy with no borrowing, respectively.[6] First, we note that the tighter limits in the economy with no borrowing imply higher precautionary savings and therefore a higher aggregate capital stock and a lower interest rate. This is due to the fact that risk sharing is more limited with no borrowing, since the endogenous limits

---

[5] If we consider individuals with zero net financial wealth as agents in debt in the data, then the proportion of individuals in debt rises to 24.31%, which is closer to the pecentage of people in debt generated by our benchmark model.

[6] Since the government expenditure $G$ is kept constant across the two economies, we have to adjust the labor taxes to make sure that the government's budget is satisfied. In this case we have to adjust $\kappa_2$ from 1.05 to 1.07.

are considerably looser than the fixed limits of zero. We also see that the economy with endogenous limits generates a higher wealth inequality, as shown by the higher coefficient of variation of the asset distribution. Moreover, in the steady state wealth distribution that is displayed in Table 2, we see that almost 1/3 of individuals are in debt when the borrowing limits are endogenous. These last two facts explain why there are significant long run welfare gains, which are of the order of magnitude of 6% in consumption equivalent terms, from tightening the limits.

Table 3: Steady State of the Benchmark Economy

|  | Benchmark Economy with Endogenous Limits | Economy with no Borrowing |
|---|---|---|
| $K$ | 5.20 | 5.42 |
| $\frac{K}{Y}$ | 3.03 | 3.11 |
| $r\%$ | 3.86 | 3.56 |
| $cv_k$ | 2.65 | 2.54 |
| $(\boldsymbol{\tau}_k, T_l)$ | $(0.4, 0.211)$ | $(0.4, 0.214)$ |
| $\frac{G}{Y}$ | 0.17 | 0.167 |
| $W$ | 100 | 106.00 |

The endogenous limits in the benchmark economy are displayed in Figure 2. The left panel of the figure shows the level of the endogenous borrowing limits as a function of income, while the right panel plots the limits as a proportion of income against income. The first observation, is that the limits get looser with income. These findings show that results that limits get looser with income is robust to the presence of progressive income taxation and a labor-leisure choice. As under proportional taxes, the limits as a proportion of income get tighter with a higher income, at least for low income levels. On the other hand, for higher income levels, the model can capture the fact that limits are relatively constant as a proportion of income. Ábrahám and Carceles-Poveda (2007c) show that these properties hold in the US data on credit limits.

Figure 2: Limits as a Function of Labour Income in the Model



Limits



Limits as a Proportion of Income

13
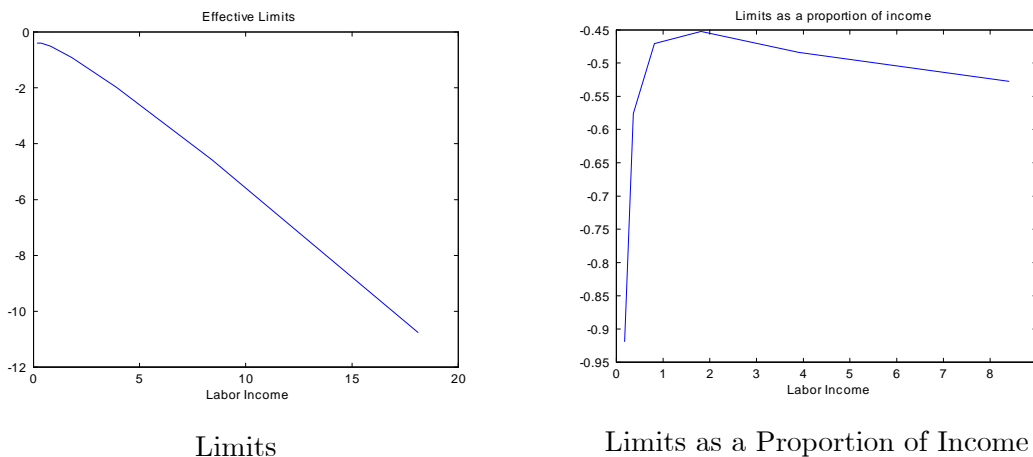
## 4. Steady State Effects of Capital Income Taxation

This section analyzes the long run welfare implications of different revenue neutral tax reforms that eliminate the capital income tax at the expense of higher labor income taxes. We first study the impact of these reforms in the presence of a fixed no short selling constraint. The results for this case are displayed in Table 4.

Table 4: Tax Reform with Exogenous Limits

| $\tau_k$ | $(\kappa_0, \kappa_1, \kappa_2)$ | $K$ | $\frac{K}{Y}$ | $L$ | $r\%$ | $r(1-\tau_k)$ | $w$ | $W$ |
|---|---|---|---|---|---|---|---|---|
| 0.4 | $(0.25, 0.76, 1.07)$ | 5.42 | 3.11 | 0.45 | 3.56 | 2.14 | 2.46 | 100 |
| 0 | $(0.36, 0.76, 1.07)$ | 5.40 | 3.20 | 0.43 | 3.23 | 3.23 | 2.50 | 100.22 |
| 0 | $(0.51, 1.536, 1.07)$ | 4.49 | 2.96 | 0.40 | 4.13 | 4.13 | 2.39 | 106.10 |
| 0 | $(0.25, 0.76, 11.94)$ | 6.16 | 3.29 | 0.44 | 2.59 | 2.59 | 2.59 | 92.58 |

The first row of the table reports the pre-reform steady state and the other three rows display the post-reform steady state under three different reforms that eliminate capital income taxes. Since we study a revenue neutral tax reform, the increase in total labor income taxes is the same across the three cases, which differ in the labor income tax code changes. Under the first reform, budget balance for the government is achieved through an increase in $\kappa_0$. Note that this implies a higher average tax rate for all income levels. While this is also true under the second reform, the progressivity of labor income taxes is also increased in this case through a higher $\kappa_1$. In turn, this requires a higher $\kappa_0$ than under the first reform to balance the government budget. Finally, the third reform balances the budget by simply increasing $\kappa_2$, while it leaves $\kappa_0$ and $\kappa_1$ unchanged. In practice, this reform flattens out the labor tax.

The first five columns display the capital income tax rate, the parameters of the labor income tax function, the capital, the capital output ratio and the labor; the next three columns display the interest rate before and after taxes and the aggregate wage rate; finally the last column displays the aggregate welfare in consumption equivalent terms. The table reflects that eliminating capital income taxes improves considerably aggregate welfare when this is done through an increase in $\kappa_1$ and $\kappa_0$. In contrast, welfare is only slightly better when only $\kappa_0$ is changed and it decreases considerably when $\kappa_2$ is increased. These results can be explained as follows.

Consider first the reform that changes only $\kappa_2$. As the table reflects, this parameter has to increase from 1.07 to 11.97 to balance the government budget in the absence of changes in the average tax rate parameter $\kappa_0$. In turn, this implies that the tax function becomes practically linear, making labor income taxes less progressive. The effects of these changes are twofold. On the one hand, agents face a higher after tax return on assets which increases both their disposable income and their incentives to save. This leads to a higher aggregate capital stock. This effect is obviously stronger for those who have assets and thus who save more (the high asset and/or high income agents). On the other hand, the change in the tax system hurts mostly the poor who have to pay a similar tax rate as the rich after the reform, while the tax rate of the rich is mostly unaffected. In other words, the reform

14

hurts considerably the poor agents, who rely mostly on labor income, by decreasing their disposable income. Our results illustrate that the negative effects dominate overall and the reform leads to a big welfare reduction.

Consider now the reform that increases both $\kappa_0$ and $\kappa_1$. In this case, the reform makes labor income taxes more progressive and this decreases disposable income for the rich considerably, leading to lower savings and to a lower aggregate capital stock. The decrease in capital increases the after-tax interest rate considerably with respect to the pre-reform steady state and this benefits all agents and in particular the relatively rich, for whom capital income is relatively more important. Obviously, the increase in progressivity also benefits the poor, who rely mostly on labor income and the tax burden of the poorest may even decrease. Overall, these positive effects offset the negative effect of a higher average labor income tax (the increase in $\kappa_0$) for everyone.

Finally, we see a very small positive effect of an increase in $\kappa_0$. In this case, we have similar effects to the ones before but much smaller. We therefore conclude that the elimination of capital income taxes will have the best chance to become a successful reform if the increase in average labor income taxes is accompanied by an increase in the progressivity.

Since the results that we have just discussed assume an exogenous borrowing constraint that is equal to zero, they abstract from the effect that a change in tax policy can have on the relative value of default and consequently on the borrowing constraints. Furthermore, a different new level of capital will also affect the borrowing constraints and the ability to self-insure indirectly. In sum, the optimal level of capital and thus the optimal capital income taxation might crucially depend on how the borrowing limits are modelled.

To evaluate the importance of these effects quantitatively when the borrowing limits also change as a response to the tax reform, we study the same revenue neutral tax reform in the presence of endogenous borrowing limits. As mentioned earlier, we assume that only agents with positive assets pay capital income taxes and we study the two reforms that increase welfare with exogenous borrowing constraints. Table 5 reports the results of the reform for these cases. In addition, the level of the endogenous limits is displayed in Table 6.

Table 5: Tax Reform with Endogenous Limits

| $\tau_k$ | $(\kappa_0, \kappa_1, \kappa_2)$ | $K$ | $\frac{K}{Y}$ | $L$ | $r\%$ | $r(1-\tau_k)$ | $w$ | $W$ |
|---|---|---|---|---|---|---|---|---|
| 0.4 | $(0.25, 0.76, 1.05)$ | 5.20 | 3.03 | 0.45 | 3.86 | 2.32 | 2.42 | 100 |
| 0 | $(0.375, 0.76, 1.05)$ | 5.15 | 3.12 | 0.42 | 3.52 | 3.53 | 2.46 | 99.60 |
| 0 | $(0.534, 1.53, 1.05)$ | 4.26 | 2.89 | 0.39 | 4.45 | 4.45 | 2.36 | 106.17 |

The first row of the table displays the pre-reform steady state and the last two rows display the post-reform steady states for the case in which only $\kappa_0$ changes to balance the government budget (row 2) and the case in which both $\kappa_0$ and $\kappa_1$ change (row 3). As we see, the results under these two reforms are relatively similar to the ones under fixed limits. Aggregate capital drops in both cases. The main reason is that the tax reform reduces the disposable income of the rich, who are the predominant savers in this economy. The increased after tax return on asset accumulation does not offset this effect. The key difference is that

the reform that increases the linear component of the tax system now leads to a welfare reduction. There are two main differences with respect to the fixed limit case that generate this result. The first one is the presence of borrowers who did not pay the capital tax either before or after the reform. The second is that the endogenous borrowing limits adjust as response to the tax reform. Below, we discuss the effect of these new elements.

Regarding the presence of borrowers, note that they face a lower interest rate after the first reform, while they face a higher rate under the second. This makes default less desirable in the first case, in which case the limits become looser, while it makes it more desirable in the second case, in which case the limits become tighter. While loosening the limits may benefit agents in the long run, it also results in the poorest agents being more indebted and therefore poorer in the new steady state. As we see, this latter negative effect is just large enough to overturn our findings with fixed limits, implying that the reform leads to a decreasing (aggregate) welfare in the long run.

Exactly the opposite happens in the second reform. Since the limits become tighter, this has benefits in the long run, as the poorest individuals in the economy are now richer. Note, however, that these agents maybe worse off during the transition, since they will have to reduce their debt to reach the new steady state, an effect that may decrease their welfare significantly.

Table 6: Effective Limits

| Benchmark | $(\kappa_0, \kappa_1, \kappa_2)$ | $\kappa(\epsilon_1)$ | $\kappa(\epsilon_2)$ | $\kappa(\epsilon_3)$ | $\kappa(\epsilon_4)$ | $\kappa(\epsilon_5)$ | $\kappa(\epsilon_6)$ | $\kappa(\epsilon_7)$ |
|---|---|---|---|---|---|---|---|---|
| $\tau_k = 0.4$ | $(0.25, 0.76, 1.05)$ | -0.402 | -0.402 | -0.506 | -0.928 | -1.987 | -4.579 | -10.75 |
| $\tau_k = 0$ | $(0.375, 0.76, 1.05)$ | -0.424 | -0.424 | -0.519 | -0.888 | -1.807 | -4.029 | -9.302 |
| $\tau_k = 0$ | $(0.534, 1.53, 1.05)$ | -0.357 | -0.357 | -0.454 | -0.769 | -1.371 | -2.798 | -6.334 |

Given this, our analysis does not provide with a final answer regarding the desirability of the tax reform we consider due to the fact that we do not take into account the transition towards the new steady state. This is an important issue that we analyze in the next section.

## 5. TRANSITIONAL EFFECTS OF CAPITAL INCOME TAXATION

In this section, we analyze the transitional effects of eliminating capital income taxes. We focus on the reforms that increase $\kappa_0$ and $\kappa_1$, since these are the ones that have a chance of having positive welfare effects.

**5.1. Transition with Fixed Limits.** We start by analyzing the case with fixed borrowing limits and the reform that increases progressivity, since this is the one that has a higher chance of having positive welfare effects. In this reform, the parameter $\kappa_1$ is increased once and for all and the parameter $\kappa_0$ adjusts every period to ensure government budget balance. The transitional path for some of the key aggregate variables is displayed in Figure 3. As we see in the figure, capital experiences a smooth decrease throughout the transition. This is due to the fact that an increase in progressivity decreases the disposable income and therefore the savings of the rich. Figure 3 also reflects that the decrease in capital is accompanied by a decrease in aggregate labor supply, reflecting again the individual labor supply responses

to the increasing labor income taxes. These changes imply that aggregate welfare increases on impact.

Figure 3: Aggregate paths with exogenous limits (increase in $\kappa_0$ and $\kappa_1$)
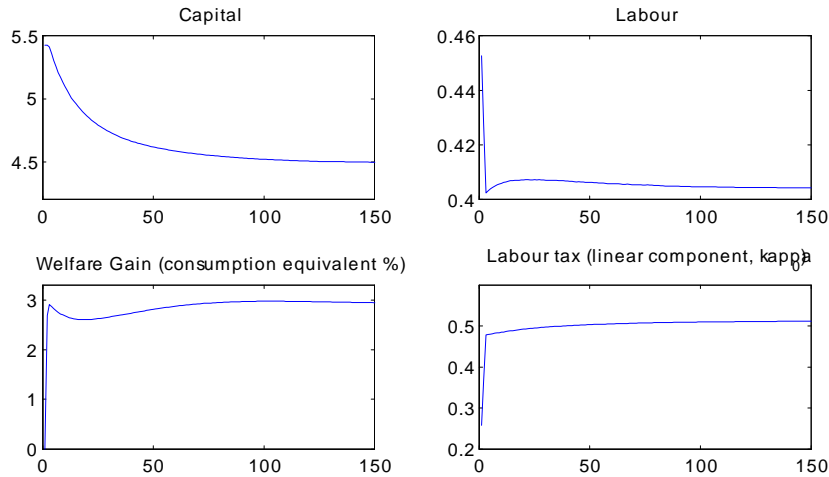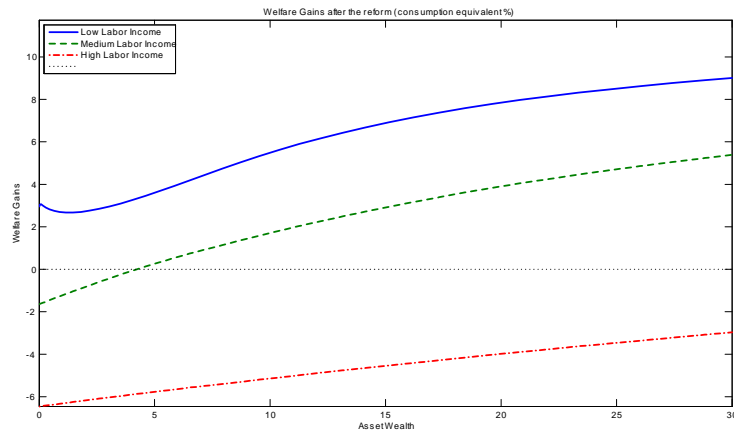


Figure 4 displays the welfare gains due the reform in consumption equivalent terms for individuals with different income shocks and asset levels.

Figure 4: Welfare gains with exogenous limits (increase in $\kappa_0$ and $\kappa_1$)



This figure is important for two reasons. It shows who are the agents who would be in favour and against the reform. Also, it indicates whether this reform could have public support or not. The answer to the first question is the following. The higher is the asset wealth of a given individual, the more he/she prefers the reform. This is not surprising, as agents with a higher asset wealth benefit more from the immediate increase of the after tax interest rate. In addition, we see that the higher is the labor income of a given individual,

the less he/she will favour the reform, since the increased progressivity in labor income taxes will hurt them the most. Overall these results imply that low income individuals will be uniformly in favour of the reform, while high income individuals will be uniformly against it. We find that the overall support is 95 percent (recall from Table 1 that the top two income groups constitute only 3 percent of the population).
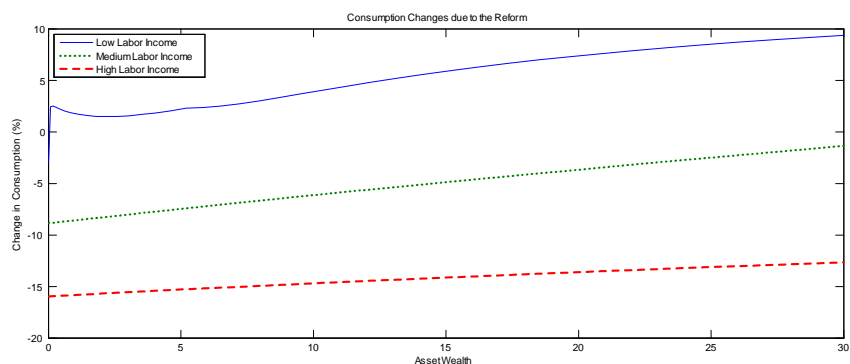
Figure 5: Individual Policy Changes with exogenous limits, (increase in $\kappa_0$ and $\kappa_1$)
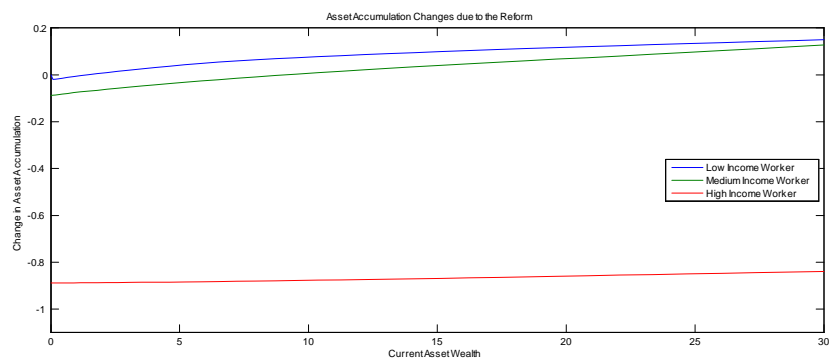


Asset Accumulation Changes
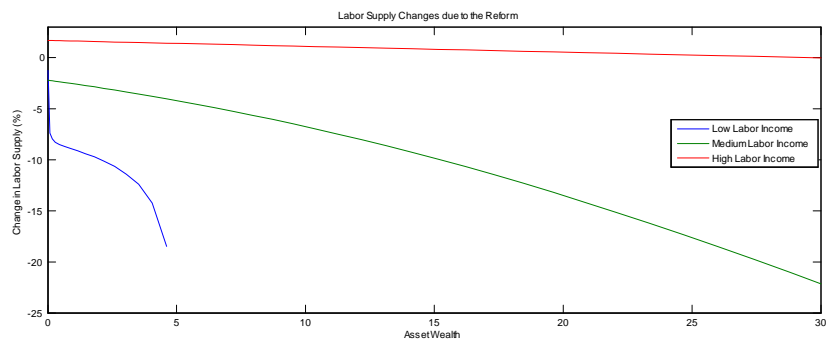


Labor Supply Changes



Consumption Changes

Finally, Figure 5 helps us understand why the welfare of different individuals changes in the way described above. The first panel shows the change in capital accumulation due to the reform for individuals with different asset wealth and income. The second and third panels show the same for the individual labor supply and consumption in percentage terms. Consider first the low income agents. As we see, they are able to afford a higher consumption and asset accumulation (except the ones with a very low asset wealth) in spite of the fact that they decrease their labor supply. The main reason is the fact that they experience a favorable tax change, since the increase in $\kappa_0$ is more than compensated by the increase in progressivity. All these changes lead to an increase in their welfare. Consider now the high income agents. After the reform, they have to reduce consumption and asset accumulation even though they have to increase their labor supply. The reason is that the tax reform hits them badly by increasing both the average tax rate on labor and the progressivity of the labor income tax. The increase in their asset income due to the increase in the after tax interest rate can then be offset only partially in their case.

These figures also help us understand better the dynamics of the aggregate variables. First, the very poor save a bit more but the rest of the population save less after the reform, leading to lower capital accumulation. Second, only the high income agents increase their labor supply. Since they only represent a small fraction of the population, however, aggregate labor supply drops. Finally, it is clear from the previous results that this economy achieves a higher welfare not through a higher aggregate output and capital accumulation but through a reduction in consumption and wealth inequality and an increase in the leisure inequality. It may thus seem that the elimination of capital income taxes plays a minor role, but it has an important effect because it induces the low to medium labor income individuals to save more.

For comparison, we also analyze the reform that keeps the progressivity parameter $\kappa_1$ constant and just adjusts the average tax rate parameter $\kappa_0$ to achieve government balance budget. We plot the same variables as before in figures 6 to 8.

Figure 6: Aggregate paths with exogenous limits (increase in $\kappa_0$)
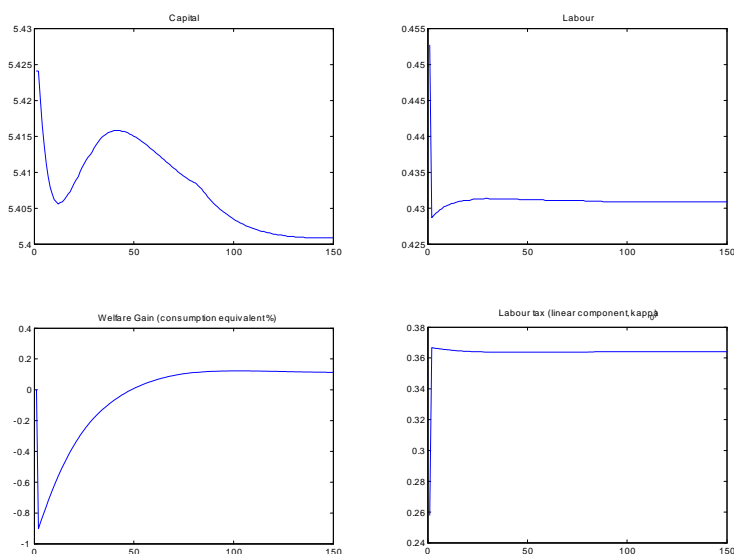


As in the previous case, figure 6 illustrates that the reform leads to a decrease in the aggregate capital and labor supply, but the magnitude (especially in the case of aggregate capital) is much smaller. Most importantly, although the reform is welfare enhancing in the long run, it reduces welfare when the transition is taken into account. Figure 7 shows why this is the case.

Figure 7: Post-reform welfare with exogenous limits (increase in $\kappa_0$)
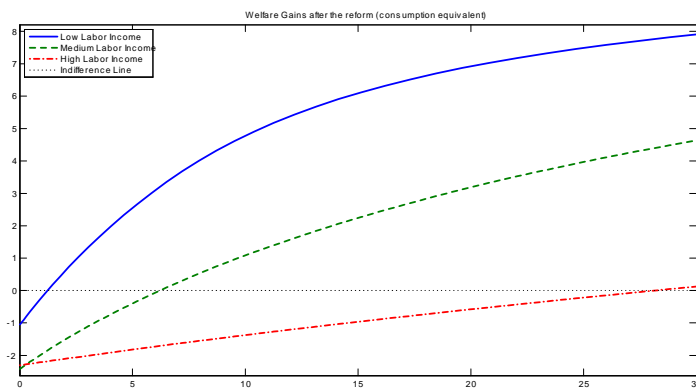


As before, asset wealth and labor income (to the most extent) determine the preference for the reform in the same way. The difference is that this reform does not hurt the high labor income agents as much as before, since the progressivity of the tax system is kept

constant. Hence, if these agents have sufficiently high asset wealth, they will actually prefer the reform. On the other hand, low income people with low assets are clearly against the reform because they are hurt considerably by the increase in taxation. Eventually, only 20 percent of the population is in favour of the reform (note that low income people are largely concentrated around the borrowing limit in the initial steady state).

Figure 8 sheds some more light on these welfare changes. As we see, low asset agents have to reduce their consumption for all income levels under this second reform. Further, the poorest individuals have to increase their labor supply to be able to support this level of consumption. The reason is that the increase in the average tax rate reduces their disposable income significantly, especially if they are not benefiting from the higher after tax interest rate. On the other hand, the rich and medium income households can afford to reduce their labor supply. Overall, this leads to a smaller decline in total labor. Note that the key reason for the failure of this reform is that it hurts the asset poor too much at impact, in the sense that they have to pay too much for financing the reform.
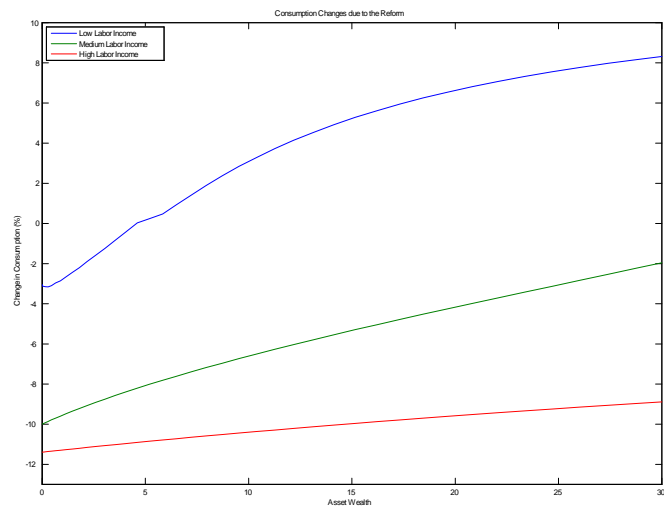
Asset Accumulation Changes



Labor Supply Changes



Consumption Changes

22
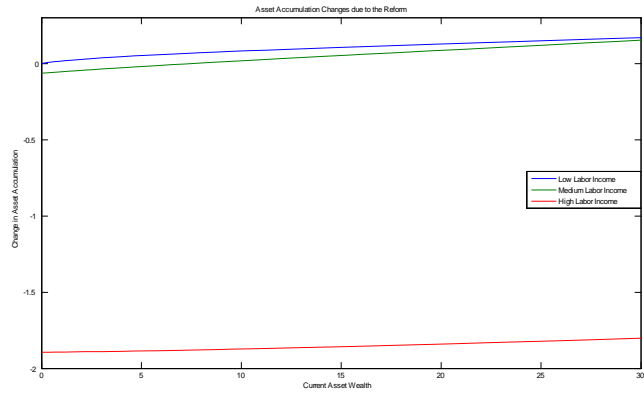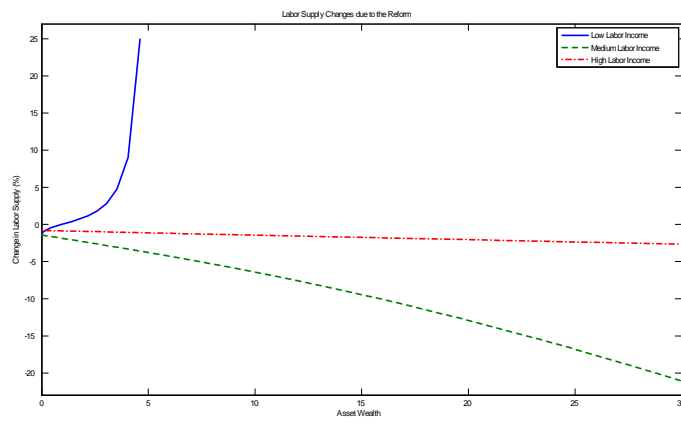
**5.2. Transition with Endogenous Limits.** To be added.

## 6. Conclusions

The present work studies whether eliminating capital taxes is desirable or not in economy with incomplete markets, capital accumulation and the possibility of default on financial liabilities. In particular, we study competitive equilibria where the loosest possible limits that prevent default are imposed.

We first calibrate the model to match the distribution of financial assets (and unsecured) debt in the US economy. Then, we analyze the welfare implications of a revenue neutral tax reform that eliminates capital income taxes at the expense of higher labor income taxes. Our benchmark economy has a progressive labor income tax code and we study different reforms that increase the average tax rate and/or modify the progressivity of the tax system.

In a setting where limits are exogenously fixed at zero, we find that an increase in aggregate welfare requires that the increase in average labor income tax rates is accompanied by an increase in the progressivity of the tax system. In this case, aggregate capital is lower in equilibrium due to the reduction in savings of high income agents, who are actually against the reform. In contrast, low to middle income households support the reform, since they have a higher disposable income and can afford a higher consumption, leisure and asset accumulation after the reform. It is important to note that aggregate welfare under this reform is higher both in the long run and throughout the transition. Moreover, the reform still leads to a higher aggregate welfare when the limits are endogenous, in which case they become tighter after the reform.

For comparison, we study two other reforms. The first keeps the average level of taxes constant but it balances the government budget with a parameter that makes the tax system practically linear. The effect of this is to make taxes less progressive, leading to long run aggregate welfare losses, as less progressivity hurts low income households more than they benefit in the the long run due to higher pre-tax wages. In addition, we also study a reform that leaves the level of progressivity unchanged but increases the average level of taxes. In this case, welfare is lower for the poor, since they have to decrease both their consumption and leisure, leading to a decrease in aggregate welfare when the transition is taken into account. Even though the long run welfare gain is slightly positive when the borrowing limits are exogenous, this effect is outweighed by the fact that more people become indebted in the long run with endogenous limits.

## APPENDIX
## Proof of Proposition 2.1

First, since the periodic utility function $u(c, 1-l)$ is continuous in consumption, $W(\epsilon, k; \Psi)$ satisfies the same property in $k$. It is also clear that the value function has to be increasing in $k$, since everything that is feasible under a given $(\epsilon, k; \Psi)$ has to be feasible under $(\epsilon, \widetilde{k}; \Psi)$ for all $\widetilde{k} \geq k$. This implies that $W(\epsilon, \widetilde{k}; \Psi) \geq W(\epsilon, k; \Psi)$ for $\widetilde{k} \geq k$. Let $\underline{k}^N(\epsilon, \Psi) < 0$ be the appropriately defined natural borrowing limit. This limit is defined as the level of debt

such that households are just able to repay their debt under every possible contingency and still have non-negative consumption.[7] Further, define the (possibly infinite) supremum of the utility function as follows: $\overline{U} \equiv \sup_{c \in \mathbb{R}_+} u(c, 1) \leq \infty$. Then, the strict monotonicity of the period utility function implies that:

$$\lim_{k \to \underline{k}^N(\epsilon, \Psi)} W(\epsilon, k; \Psi) = -\infty \text{ and } \lim_{k \to \infty} W(\epsilon, k; \Psi) = \frac{\overline{U}}{1 - \beta}.$$

The first equality follows from the fact that $u$ is unbounded below and from the fact that, at the natural borrowing limit, households would end up consuming zero along some history with positive probability. In addition, since our assumptions imply that the shock $\epsilon$, the labor supply $l$ and the aggregate capital $K$ are positive and finite, it also follows that $-\infty < V(\epsilon; \Psi) < \frac{\overline{U}}{1-\beta}$. Given this, the intermediate value theorem implies that there exists a finite $\underline{k} > \underline{k}^N(\epsilon, \Psi) > -\infty$ for $\forall(\epsilon, \Psi)$ such that $W(\epsilon, \underline{k}; \Psi) = V(\epsilon; \Psi)$. Further, the uniqueness of $\underline{k}$ follows from the fact that $W$ is increasing in $k$.

Second, since $W(\epsilon, \underline{k}; \Psi) = V(\epsilon; \Psi)$ and $W(\epsilon, 0; \Psi) \geq V(\epsilon; \Psi)$, the fact that $W$ is increasing in $k$ implies that $\underline{k}(\epsilon; \Psi) \leq 0$ and therefore $\sup_{(\epsilon; \Psi)} \{\underline{k}(\epsilon; \Psi)\} \leq 0$. In other words, the equilibrium default threshold and effective limits are non-positive. ∎

## REFERENCES

[1] Ábrahám, Á. and E. Cárceles-Poveda (2007a). "Endogenous Incomplete Markets, Enforcement Constraints and Intermediation", *Theoretical Economics*, 1-4, 2006, pp. 439–459.

[2] Ábrahám, Á. and E. Cárceles-Poveda (2007b). "Competitive Equilibria with Production and Limited Commitment", Unpublished Manuscript, University of Rochester and SUNY Stony Brook.

[3] Ábrahám, Á. and E. Cárceles-Poveda (2007c). "Endogenous Trading Constraints in Incomplete Asset Markets"", Unpublished Manuscript, University of Rochester and SUNY Stony Brook.

[4] Aiyagari, D. R. (1994). "Uninsured Idiosyncratic Risk and Aggregate Saving", *The Quarterly Journal of Economics* 109(3), pp. 659-684.

[5] Aiyagari, D. R. (1995). "Optimal Capital Income Taxation with Incomplete Markets, Borrowing Constraints and Constant Discounting ", *The Journal of Political Economy* 103(6), pp. 1158-75.

[6] Alvarez, F. and U. Jermann (2000). "Efficiency, Equilibrium, and Asset Pricing with Risk of Default", *Econometrica* 68(4), pp. 775-797.

[7] Alvarez, F. and U. Jermann (2001). "Quantitative Asset Pricing Implications of Endogenous Solvency Constraints", *Review of Financial Studies* 14(4), pp. 1117-1151.

---

[7] See Santos and Woodford (1997) for a specification of such limits in a general incomplete markets context. As shown by the authors, the natural limits represent an appropriately defined present value of the individual labor income, which has to be finite in equilibrium.

[8] Bai, Y. and Zhang, J. (2006). " Solving the Feldstein-Horioka Puzzle with Financial Frictions ", Unpublished Manuscript, Arizona State University and University of Michigan.

[9] Castaneda A., Diaz Gimenez, J. and J. V. Rios-Rull (2003). "Accounting for earnings and wealth inequality", *Journal of Political Economy,* 111(4), pp. 818-857.

[10] Chamley, C. (1986). "Optimal Taxation of Capital Income in General Equilibrium with Infinite Lives", *Econometrica* 54(3), pp. 607-622.

[11] Davila, J., Hong, J. H., Krusell, P. and J. V. Rios-Rull (2007). "Constrained Efficiency in the Neoclassical Growth Model with Uninsurable Idiosyncratic Shocks", Unpublished Manuscript, CERMSEM, University of Pennsylvania and Princeton University.

[12] Domeij D. and J. Heathcote (2004). "On the Distributional Effects of Reducing Capital Taxes", *International Economic Review*, 45(2), pp. 523-554.

[13] Heaton, J. and D. Lucas (1996). "Evaluating the Effects of Incomplete Markets on Risk Sharing and Asset Pricing", *Journal of Political Economy* 104(3), pp. 443-487.

[14] Huggett, M. (1997). "The One-Sector Growth Model with Idiosyncratic Shocks: Steady States and Dynamics", *Journal of Monetary Economics* 39, pp. 385- 403.

[15] Judd, K. (1985). "Redistributive Taxation in a simple Perfect Foresight Model", *Journal of Public Economics* 28, pp. 59-83.

[16] Kehoe, P., and F. Perri (2002). "International Business Cycles with Endogenous Incomplete Markets", *Econometrica*, 70(3), pp. 907-928.

[17] Kehoe, P., and F. Perri (2004). "Competitive Equilibria with Limited Enforcement", *Journal of Economic Theory*, 119(1), pp. 184-206.

[18] Kehoe, T., and D. Levine (1993). "Debt Constrained Asset Markets", *Review of Economic Studies* 60(4), pp. 865-888.

[19] Krueger, D., and F. Perri (2005). "Public versus Private Risk Sharing", Unpublished Manuscript, University of Pennsylvania and Minneapolis Fed.

[20] Krueger, D., and F. Perri (2005a). "Does Income Inequality lead to Consumption Inequality?, Evidence and Theory", *Review of Economic Studies* 73.

[21] Krueger, D., and F. Perri (2005b). "On the Welfare Consequences of the Increase in Inequality in the United States", *NBER Macro Annual*, pp. 83-121.

[22] Krusell, P., and A. Smith (1997). "Income and Wealth Heterogeneity, Portfolio Choice, and Equilibrium Asset Returns", *Macroeconomic Dynamics*, 1(2), pp. 387-422.

[23] Krusell, P., and A. Smith (1998). "Income and Wealth Heterogeneity in the Macroeconomy", *Journal of Political Economy*, 106(5), pp. 867-896.

[24] Livshits, MacGee and Tertirlt (2007). "Accounting for the Rise in Consumer Bankruptcies in the United States", Unpublished Manuscript, University of Western Ontario.

[25] Mendoza, E., Razin A. and L. Tesar (1994). "Effective Tax Rates in Macroeconomics. Cross Country Estimates of Tax Rates on Factor Income and Consumption", *Journal of Monetary Economics* 34, pp. 297-323.

[26] Santos, M., and M. Woodford (1997). "Rational Asset Pricing Bubbles," *Econometrica* 65(1), pp. 10-57.

[27] Sullivan, T., E. Warren, and J. L. Westbrook (1989). "As We Forgive Our Debtors", Oxford University Press, Oxford, New York.

[28] Telmer, C. (1993). "Asset Pricing Puzzles and Incomplete Markets", *The Journal of Finance* 48(5), pp. 1803-1832.

[29] Zhang, H. (1997a). "Endogenous Borrowing Constraints with Incomplete Markets", *Journal of Finance* 52 (5), pp. 2187-2209.

[30] Zhang, H. (1997b). "Endogenous Short Sale Constraint, Stock Prices and Output Cycles", *Macroeconomic Dynamics* 1(1), pp. 228-254.